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EXISTENCE OF THREE SOLUTIONS FOR A P-BIHARMONIC PROBLEM

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Abstract. In this paper we study the multiplicity results for a p-biharmonic problem. The existence of an open interval of parameters which ensures this problem admits at least three solutions is determined by a variational method of G. Bonanno.

Keywords. Three solutions, p-biharmonic equation, fourth-order problem.

1 Introduction

In this paper, we assume that $\Omega \in R^n$ is a nonempty bounded open set with C^2 boundary $\partial\Omega$, $2p > n$, $\lambda > 0$ and $f : \Omega \times R \to R$ is a Carathéodory function.

We consider the multiplicity theorem for the p-biharmonic problem

$$\begin{cases} \Delta(|\Delta u|^{p-2}\Delta u) - \gamma\, div(|\nabla u|^{p-2}\nabla u) = \lambda f(x,u) & in \quad \Omega, \\ u = \frac{\partial \Delta u}{\partial \nu} = 0 & on \quad \partial\Omega, \end{cases} \tag{1}$$

where $\gamma > 0$ is a constant. And for convenience, we firstly consider the so-called autonomous case, i.e,

$$\begin{cases} \Delta(|\Delta u|^{p-2}\Delta u) - \gamma\, div(|\nabla u|^{p-2}\nabla u) = \lambda f(u) & in \quad \Omega, \\ u = \frac{\partial \Delta u}{\partial \nu} = 0 & on \quad \partial\Omega, \end{cases} \tag{2}$$

Particularly, in the case of $p = 2$, Problem (1), (2) reduce to the following bi-harmonic equation

$$\begin{cases} \Delta^2 u - \gamma \Delta u = \lambda f(x,u) & in \quad \Omega, \\ u = \frac{\partial \Delta u}{\partial \nu} = 0 & on \quad \partial\Omega. \end{cases} \tag{3}$$