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ON STABILITY OF DIFFERENTIAL EQUATIONS WITH PIECEWISE CONSTANT ARGUMENT AND THE ASSOCIATED DISCRETE EQUATIONS USING DICHOTOMIC MAP

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Abstract. Dichotomic maps are considered by means of the stability of the null solution of a class of differential equations with piecewise constant argument via associated discrete equations.

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1 Introduction

The study of differential equations with piecewise continuous argument has been the subject of some investigations such as Aftabizadeh & Wiener [1], Carvalho & Cooke [3], Carvalho & Ferreira [4], Carvalho & Marconato [5], Cooke & Wiener [6,7,8], Marconato [9,11] and Marconato & Spezamiglio [10]. Some equations of this type are similar in structure to those found in certain "sequential-continuous" models of disease dynamics as treated by Busenberg & Cooke [2].

Those equations include, as a particular case, the differential equations with argument constant in intervals, such:

$$x'(t) = f(t, x(t), x([t])) \tag{1.1}$$

where $f : \mathbb{R} \times \mathbb{R}^n \times \mathbb{R}^n \to \mathbb{R}^n$ is a continuous map with $f(t, 0, 0) = 0$ for all $t \in \mathbb{R}$.

We further impose that f takes bounded sets into bounded sets and satisfies enough additional smoothness conditions to ensure the existence, uniqueness and continuous dependence with respect the initial conditions, of the solutions of (1.1). The retarded of the equation (1.1) is sectionally continuous because $r(t) = t - [t]$ is a continuous map in $[n, n+1)$, for all $n \in \mathbb{Z}$, and discontinuous at integers values.

It is possible to obtain recurrence relations for the expressions of the solutions of (1.1) in the intervals between integers, and then, to determine a discrete equation of the type