*Dynamics of Continuous, Discrete and Impulsive Systems*
*Series* **B:** *Applications & Algorithms* **15 (2008)** 165-178


# EXTINCTION IN TWO DIMENSIONAL DISCRETE LOTKA-VOLTERRA COMPETITIVE SYSTEM WITH THE EFFECT OF TOXIC SUBSTANCES

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**Abstract.** In this paper, we consider a nonautonomous discrete Lotka-Volterra system of two species with the effect of toxic substances. We prove that one of the components will be driven to extinction while the other will be globally attractive with any positive solution of a discrete logistic equation under some conditions. For periodic case, we also obtain another set of sufficient conditions which guarantee the extinction and the existence of a globally stable periodic solution. It is shown that toxic substances play an important role in the extinction of species.

**Keywords.** Discrete; Toxicology; Extinction; Periodic solution; Global stability.
**AMS (MOS) subject classification:** 34C05, 34C25.

# 1 Introduction

In [1], Shair Ahamd considered the following nonautonomous system of differential equations:

$$\begin{aligned} x_1'(t) &= x_1(t)\big[r_1(t) - a_{11}(t)x_1(t) - a_{12}(t)x_2(t)\big], \\ x_2'(t) &= x_2(t)\big[r_2(t) - a_{21}(t)x_1(t) - a_{22}(t)x_2(t)\big], \end{aligned} \tag{1.1}$$

where $r_i(t)$ and $a_{ij}(t)$ are all continuous functions which are bounded above and below by positive constants. Ahmad [1] extended the *principle of competitive exclusion* from autonomous systems to nonautonomous systems for two species, that is, under some algebraic inequalities, there can be no coexistence of the two species, one of them will be driven to extinction while the other will stabilize at a certain solution of a logistic equation. For more works on this direction, one could refer to [3,4,11,12] and the references cited therein.

If we further assume that each species produces a toxic substance to the other, but only when the other is present, then, a suitable system can be written as

$$\begin{aligned} x_1'(t) &= x_1(t)\big[r_1(t) - a_{11}(t)x_1(t) - a_{12}(t)x_2(t) - b_1(t)x_1(t)x_2(t)\big], \\ x_2'(t) &= x_2(t)\big[r_2(t) - a_{21}(t)x_1(t) - a_{22}(t)x_2(t) - b_2(t)x_1(t)x_2(t)\big], \end{aligned} \tag{1.2}$$