

## SADDLE-NODE/MULTIPLE HOPF INTERACTION IN RETARDED FUNCTIONAL DIFFERENTIAL EQUATIONS

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**Abstract.** In this paper, we study the realizability problem for retarded functional differential equations near an equilibrium point undergoing a nonlinear mode interaction between a saddle-node bifurcation and a non-resonant multiple Hopf bifurcation. In contrast to the case of transcritical/multiple Hopf interaction which was studied in an earlier paper [4], the analysis here is complicated by the presence of a nilpotency which introduces a non-compact component in the symmetry group of the normal form. We present a framework to analyse the realizability problem in this non-semisimple case which exploits to a large extent our previous results for the realizability problem in the semisimple case. Apart from providing a solution to the problem of interest in this paper, it is believed that the approach used here could potentially be adapted to the study of the realizability problem for toroidal normal forms in the general case of repeated eigenvalues with Jordan blocks.

**Keywords.** Delay-differential equations, bifurcation, realizability, symmetry, normal forms.

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### 1 Introduction

Retarded functional differential equations (RFDEs) are frequently used as models for various phenomena [1, 2, 16, 17, 18, 19, 20]. While the phase space for the resulting dynamical system is infinite dimensional, the existence of finite-dimensional invariant center manifolds near bifurcation points imply that much of the machinery developed for the analysis of finite codimension bifurcations in ordinary differential equations (e.g. normal forms, unfolding theory) are portable to RFDEs. Indeed, there is ample evidence in the literature, e.g. [1, 16, 19], that these tools and techniques of local analysis can give much valuable information about the dynamics of RFDEs.

In this context, one of the fundamental questions concerns the characterization of the range of dynamics accessible near a bifurcation point for a given RFDE model. This is not a trivial question, since even for scalar RFDEs, it is possible to have bifurcations of equilibria with large dimensional center manifolds. In this case, if there are not enough independent delay terms