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EXISTENCE AND UNIQUENESS OF SOLUTIONS FOR NTH-ORDER NONLINEAR THREE-POINT BOUNDARY VALUE PROBLEMS

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Abstract. In this paper, by using Leray-Schauder degree theory and Wirtinger-type inequalities, we establish the existence and uniqueness theorems for a class of nth order nonlinear three-point boundary value problems.

Keywords. Boundary value problem, Existence, Uniqueness, Leray-Schauder degree, Wirtinger-type inequalities.

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1 Introduction

In this paper, we shall consider nth-order nonlinear three-point boundary value problems(BVP)

$$u^{(n)} = f(t, u, u', \ldots, u^{(n-1)}) - e(t), \quad 0 < t < 1, \tag{1.1}$$

$$u(0) = 0, \quad u^{(i)}(\eta) = 0, i = 0, 1, 2, \cdots, n-3, \quad u(1) = 0, \tag{1.2}$$

where $f : [0,1] \times \mathbb{R}^n \to \mathbb{R}$ is a given function satisfying Carathéodory's conditions, $e : [0,1] \to \mathbb{R}$ be a function in $L^1[0,1]$, and $\eta \in (0,1)$ be given.

Motivated by the results of [5], we also refer the reader to [1-4,6], which obtained existence and uniqueness theorems for BVP(1.1),(1.2). Our method makes use of the Leray-Schauder continuation theorem and Wirtinger-type inequalities.

For obtaining the main results, the following lemmas are crucial.

Lemma 1.1 [2] *If $u(t) \in C^1[0,1]$ and $u(0) = 1$, then $\|u\|_2^2 \le \frac{4}{\pi^2}\|u'\|_2^2$.*

Lemma 1.2 [2] *If $u(t) \in C^1[0,1]$ and $u(0) = u(1) = 0$, then $\|u\|_2^2 \le \frac{1}{\pi^2}\|u'\|_2^2$.*

Lemma 1.3 [2] *Let $M_\eta = max\{\eta, 1-\eta\}$, $0 \le \eta \le 1$, if $u(\eta) = 0$, then $\|u\|_2^2 \le \frac{4}{\pi^2} M_\eta^2 \|u'\|_2^2$.*

We use classical spaces $C[0,1]$, $C^k[0,1]$, $L^k[0,1]$, and $L^\infty[0,1]$ to denote continuous, k-times continuously differentiable, measurable real functions