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ENTIRE SOLUTIONS OF CERTAIN TYPE DIFFERENTIAL EQUATIONS

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Abstract. In this paper, we deal with the problem of the existence of entire solutions of certain type differential equations using Nevanlinna theory, and answer a question proposed by C. C. Yang.

Keywords. uniqueness, value sharing, differential equations, entire solution.
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1 Introduction

By a meromorphic function we mean a function that is meromorphic in the whole complex plane. It is assumed the reader is familiar with the fundamental results of the Nevanlinna theory and its notations $T(r,f)$, $m(r,f)$, $N(r,f)$, $\bar{N}(r,f)$, $S(r,f)$ and so on, which can be found, for instance, in [5][6]. For any non-constant meromorphic function f, we denote by $S(r,f)$ any quantity satisfying

$$\lim_{r\to\infty}\frac{S(r,f)}{T(r,f)}=0,$$

possibly outside of a set of finite linear measure. Let f and g be meromorphic functions and a be a complex constant. We say that f and g share the value a IM(ignoring multiplicities), if $f-a$ and $g-a$ have the same zeros; if $f-a$ and $g-a$ have the same zeros with the same multiplicities, we say that f and g share the value a CM (counting multiplicities). Let S be a subset of distinct elements in the finite complex plane C. Define

$$E_f(S)=\bigcup\nolimits_{a\in S}\{z\mid f(z)-a=0, \textit{counting multiplicities}\}.$$

We say that $f(z)$ and $g(z)$ share the set S CM provided that $E_f(S)=E_g(S)$.

Nevanlinna's value distribution theory of meromorphic functions has been used to study the growth, oscillation, solvability and existence of entire or meromorphic solutions of linear differential equations in complex domains(see [9][10][11] etc.). It can be also used to investigate certain type of nonlinear differential equations, see, [4][7].

Recently, C. C. Yang suggested the following question during his visiting at Shandong University: Is it true that the only possible solutions of the