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STABILITY OF ROSENBROCK METHODS WITH VARIABLE STEPSIZE FOR PANTOGRAPH EQUATION [1]

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Abstract. This paper develops stability of Rosenbrock methods with variable stepsize for the pantograph equation. It is proved that the natural Rosenbrock methods are asymptotically stable if and only if the modulus of stability function at infinity is less than 1.
Keywords. Pantograph equation, Delay, Stability, Rosenbrock method, Geometric mesh.
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1 Introduction

Recently, much research has been focused on the numerical methods for a class of infinite delay differential equations with proportional time lag:

$$y'(t) = f(t, y(t), y(qt)), \quad t > 0, \tag{1}$$

where $q \in (0, 1)$, $y(0) = y_0 \in \mathcal{C}$ and $f : \mathcal{R}^+ \times \mathcal{C} \times \mathcal{C} \to \mathcal{C}$ is the given function. This system arises in a variety of scientific fields such as electric mechanics, optics and so on. A comprehensive list of applications is given in [5].

Since pantograph equation has unbounded time lags, it is usually difficult to investigate numerically the long time dynamical behavior of exact solution due to the limited computer memory as shown in [7]. In fact, a nonstationary system is yielded in its analysis of numerical stability, which is significantly more difficult than stationary difference formula (see [6]).

One way to avoid this storage problem is to transform the system into an equation with constant time lags but variable coefficients. Another way is to apply numerical method with variable stepsize. To our knowledge, most references were concerned with θ-methods or Runge-Kutta methods for this system. Few papers discuss Rosenbrock methods for pantograph equation.

Rosenbrock methods are among the easiest methods to program. They are linear and similar to Runge-Kutta methods. They are also called semi-implicit, semi-explicit, generalized, modified, additive or adaptive Runge-Kutta methods (see [3]). In this paper, we consider Rosenbrock methods with variable stepsize for the pantograph equation.

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