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AN SIR EPIDEMIC MODEL WITH NONLINEAR BIRTH PULSES

Zhen Zhang[1], Zhen Jin and Jinxiao Pan

[1]Department of Applied Mathematics
North University of China, Taiyuan, Shanxi, 030051, P. R. China

Abstract. An SIR epidemic model with logistic population dynamics and nonlinear birth pulses is considered in this paper. The basic reproductive number R_0 is defined. We obtain the exact infection-free periodic solution of the impulsive epidemic system. By using the discrete dynamical system generated by a monotone, concave map for the population, we prove that the infection-free periodic solution is globally asymptotically stable if $R_0 < 1$. We use standard bifurcation theory to show the existence of positive periodic solution if $R_0 > 1$. Numerical simulation is given in the paper.

Keywords. Dynamics; Epidemic model; Basic reproductive number; Globally stable; The infection-free periodic solution; The positive periodic solution.

AMS (MOS) subject classification: 92B05, 34A37, 34D23, 34K18.

1 Introduction and model formulation

Impulsive differential equations have been widely used to study the dynamic behavior of ecological systems and epidemic models. Compared with traditional ordinary differential equations, impulsive differential equations provide a natural description of such systems. They generally describe the phenomena which is subject to steep or instantaneous changes and can be found in almost every branch of applied sciences. For example, a fisherman may go fishing at the same time once a day or once a week; the vaccination in children aged one to several years may be given at the same time every two years; the births of some wild animals may be seasonal or occur in regular pulses and so on. In terms of the mathematical treatment, the presence of impulses gives the system a mixed nature, both continuous and discrete. The qualitative properties of the system are embodied in those of the discrete system which determines the state after a pulse in terms of the state after the previous pulse. The theory of impulsive differential equations can be found in references [1,2].

The ecological system is often deeply perturbed by impulsive effect and has received much attention from researchers. In reference [3], the researchers studied the existence of positive periodic solution of periodic time-dependent predator-prey system with impulsive effects. In [4], the authors considered the global behaviors of the periodic logistic system with periodic impulsive