

## CENTER AND ISOCHRONOUS CENTER AT INFINITY IN A CLASS OF PLANAR DIFFERENTIAL SYSTEMS

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**Abstract.** In this paper, the conditions of center and isochronous center at the infinity for a class of planar differential systems are studied. By a transformation, we first transform the infinity (the equator) of the systems into the origin. Then sufficient and necessary conditions for the infinity (the equator) of the systems being a center are obtained. A Construction Theorem of periodic constants is presented, which plays an important role in simplifying periodic constants. A complete classification of the sufficient and necessary conditions is given for the infinity of the systems being an isochronous center. All the computations for the quantities at infinity and periodic constants are performed using computer algebraic system – Mathematics 4.2, and the technique employed in this paper is different from others used in the literature.

**Keywords.** Planar system, infinity, center, isochronous center, focus value.

**AMS (MOS) subject classification:** 34C05.

## 1 Introduction

In the qualitative theory of planar differential systems, the problem to determine the conditions of center and isochronous center has received good attentions. In the case of the origin, many results have been obtained (see [27, 23, 25, 12, 2, 14]). Computation of focal values (Lyapunov constants) is one approach to study center conditions. For computing focal values, conventional methods include the method of Poincare return map and the method of Lyapunov coefficients (see [2]). In [17, 5], the authors gave a new computational method, which combined the calculation of focal values and saddle qualities into a unified calculation of singular point quantities.

For any center of a planar differential system, the largest neighborhood of the center, which is entirely covered by periodic orbits, is called the periodic annulus of the center. The function associated with the period of any periodic orbits in the periodic annulus is called periodic function. If the periodic function is a constant, the center is then said to be isochronous.