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CONSTRUCTION OF REACHABLE SETS FOR IRREVERSIBLE POSITIVE LINEAR DISCRETE DYNAMICAL SYSTEMS

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Abstract. We present an improved algorithm for the calculation of reachable sets of positive linear discrete-time systems with polyhedral initial set and polyhedral control sets (polyhedral cones, polytopes, or polyhedra). In particular, the algorithm caters for the case where the transition matrix is singular and the dynamics are irreversible. We demonstrate the algorithm with a numerical example.

Keywords. positive systems, discrete-time systems, reachable sets, reachability, controllability.

1 Introduction

Consider the positive linear discrete-time system (PLDS)

$$\mathbf{x}_{t+1} = \mathbf{A}\mathbf{x}_t + \mathbf{B}\mathbf{u}_t, \qquad t = 0, 1, 2, \dots \tag{1}$$

where $\mathbf{A} \in \mathbb{R}_+^{n\times n}$ is a non-negative transition matrix, $\mathbf{B} \in \mathbb{R}_+^{n\times m}$ is a non-negative control matrix and $\mathbf{u}_t \in \mathbb{R}_+^m$ are the non-negative control vectors. If the initial system state vector $\mathbf{x}_0 \in \mathbb{R}_+^n$, then the states $\mathbf{x}_t \in \mathbb{R}_+^n\ \forall t \geq 0$, or in words, if the initial state is non-negative, then the trajectory of the system will lie entirely in the non-negative orthant. The reader is referred to [12], [8] and [11] for introductory surveys of positive systems.

The reachable set of a PLDS at time T is the set of possible states the system may reach from a given set of allowable initial states X_0 at $t = 0$ when influenced by control vectors $\mathbf{u}_t$ lying in some specified sets Ω_t at $t = 0, 1, \dots, T-1$. The determination of the reachable sets is of particular importance when issues of accessibility and controllability are to be addressed.

The majority of existing algorithms for constructing the reachable sets of constrained linear systems have been developed essentially for polytopes (bounded sets) and cannot be adapted for polyhedral cones (unbounded sets) and general unbounded polyhedra (see [10] and the references cited therein).