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UNSATURATED POSITIVE SOLUTIONS FOR A CLASS OF NEUTRAL DIFFERENCE EQUATIONS

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Abstract. The significance of frequency measure is to reveal the oscillatory property of a sequence. Based on this concept, the definition of unsaturated positive sequence is introduced and applications are given to establish some criteria of unsaturated positive solutions for a class of neutral difference equations.

Keywords. Frequency measure; Unsaturated positive sequence; Neutral type; Difference equation; Solution.

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1 Introduction

Recall that the numerical schemes for computing solutions of the equation $g(x) = 0$. For given initial values x_{-1} and x_0, we may take a so-called discrete Newton measure to obtain the iterative formula as follows

$$x_{n+1} = x_n - \frac{g(x_n)}{g(x_n) - g(x_{n-1})}(x_n - x_{n-1}), n = 0, 1, 2, \cdots.$$

Such an equation is a particular case of the following general neutral difference equations of the form

$$\Delta(c_n^{(1)} x_n + c_n^{(2)} x_{n-k}) + f(n, x_n, x_{n-l}) = 0, n = 0, 1, 2, \cdots, \tag{1}$$

where $k \geq 1$ and $l \geq 1$ are integers, $\{c_n^{(1)}\}$ and $\{c_n^{(2)}\}$ are real sequences and $c_n^{(1)} \neq 0$ for all n, f is a real function defined on $\mathbf{Z} \times \mathbf{R}^2$.

As usual, given initial values x_i for $-\max\{k, l\} \leq i \leq 0$, we can calculate $x_1, x_2, x_3, ...$ successively in a unique manner. Such a sequence $\{x_n\}$ is said to be a solution of (1).

In general, a real sequence $\{x_n\}$ is said to be oscillatory if for every positive integer M, there are $n_1, n_2 \geq M$ such that $x_{n_1} x_{n_2} \leq 0$. For example, the sequence $x = \{1, -1, 1, -1, 1, -1, ...\}$ and $y = \{1, 1, 1, -1, 1, 1, 1, -1, ...\}$ are both said to be oscillatory. But the oscillatory frequency of x is different from y. For this sake, the concept of frequency measure is put forward in [1] (see also [5]). Following this concept, we give the definition of unsaturated positive (or unsaturated negative) sequence in section 2 and establish in