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DYNAMICS OF PERIODIC LOTKA–VOLTERRA SYSTEMS WITH GENERAL TIME DELAYS

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Abstract. The Lotka–Volterra model of interacting populations is useful in the design of competitive self-organizing systems and neural networks. This paper investigates the dynamical behavior of periodic Lotka–Volterra systems with general time delays. Some sufficient conditions independent of inputs and delays are obtained for the boundedness of trajectories and the global stability of nonnegative periodic solutions. These conditions are expressed in M-matrices and easy to verify.

Keywords. Lotka–Volterra system, neural networks, time delay, periodic solution, M-matrix.

AMS (MOS) subject classification: 34C28, 34H05, 94C15, 94B90

1 Introduction

The Lotka–Volterra model was initially proposed to describe interacting populations in an ecosystem [1]. In recent years, this model has been found useful in the design of competitive self-organizing systems used in associative memory and biocybernetics [2, 3]. The Lotka–Volterra neural networks, derived from conventional membrane dynamics of competing neurons, provided a theoretic basis for understanding neural selection mechanisms [4], and were implemented by subthreshold MOS integrated circuits [5]. Moreover, they are representative of a broader class of neural networks, namely the Cohen–Grossberg networks [6]. The dynamical properties of neural networks have attracted increasing attention due to their potential applications in classification, associative memory, optimization and signal processing [7, 8, 9, 10, 11].

The analysis of dynamical behaviors, including stability of equilibria, periodic oscillations, bifurcation and chaos, plays a key role in the study of Lotka–Volterra systems. Especially, the periodic Lotka–Volterra systems are important since the phenomena of periodicity are very common in the real world, which may be caused by seasonality of environments, oscillation of growth rates, or fluctuation of input signals. The stability of equilibria can also be treated as a special case of periodicity. Recently, the periodicity of