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ON THE UPPER BOUND OF THE NUMBER OF LIMIT CYCLES OBTAINED BY THE SECOND ORDER AVERAGING METHOD

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Abstract. For ε small we consider the number of limit cycles of the system $\dot{x} = -y(1+x)+\varepsilon F(x,y)$, $\dot{y} = x(1+x)+\varepsilon G(x,y)$, where F and G are polynomials of degree n starting with terms of degree 1. We prove that at most $2n-1$ limit cycles can bifurcate from the periodic orbits of the unperturbed system ($\varepsilon = 0$) using the averaging theory of second order under the condition that the second order averaging function is not zero.
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1 Introduction

This paper is concerned with the number of limit cycles that can bifurcate from the periodic orbits of a class of planar quadratic systems under small polynomial perturbation of degree $n \in \mathbb{N}$. We assume that the unperturbed system is the linear center with a straight line of singular points. More explicitly, we consider the two dimensional polynomial differential system

$$\begin{aligned} \dot{x} &= -y(1+x)+\varepsilon F(x,y), \\ \dot{y} &= x(1+x)+\varepsilon G(x,y), \end{aligned} \tag{1}$$

where F and G are polynomials of degree n starting with terms of degree 1. We note that system (1) for $\varepsilon = 0$ is not Hamiltonian.

One often analyze the number of limit cycles bifurcating from a center by the first return map,

$$\mathcal{P}(h,\varepsilon) - h = \varepsilon M_1(h) + \varepsilon^2 M_2(h) + \cdots + \varepsilon^k M_k(h) + \cdots ,$$

where $M_k(h)$ is called the k–order Poincaré–Pontryagin function (also called Melnikov function). If $M_k \not\equiv 0$, and $M_i \equiv 0$ for $i = 1, 2, \cdots, k-1$ in some open segments, then the maximum number of simple zeros of $M_k(h)$ give an upper bound of the number of limit cycles up to k order. For example, many