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THE METHOD OF QUASILINEARIZATION FOR SECOND ORDER FOUR POINT BOUNDARY VALUE PROBLEMS*

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Abstract. In this paper, we obtain the existence and uniqueness results of solutions for general second order four point boundary value problems by using the method of upper and lower solution together with Leray-Schauder degree, as well as obtain bilateral iteration schemes which converge quadratically to the unique solutions of the specific second order four point boundary value problems by using the method of quasilinearization.

Keywords. Four point boundary value problem; Upper and Lower solutions; Nagumo-type condition; Leray-Schauder degree; Quasilinearization; Quadratic convergence

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1 Introduction

Recently many authors have applied the method of quasilinearization to second order boundary value problems, for instance see [1, 2, 4-8, 11] and the references therein. However, the works on multipoint boundary value problems for second order differential equations by using the method of quasilinearization are quite rarely seen ([2], for example).

In this paper by applying the upper and lower solution method together with Leray-Schauder degree we shall first deal with the existence and uniqueness of solutions for the general second order four point boundary value problem

$$x'' = f(t, x, x'), \quad t \in [a, b] \tag{1.1}$$

with the four point boundary conditions

$$\varphi(x(a), x'(a)) = k(x(c)), \quad \psi(x(b), x'(b)) = g(x(d)), \tag{1.2}$$

where $f : [a, b] \times \mathbb{R}^2 \to \mathbb{R}$ is a continuous function, $\varphi, \psi : \mathbb{R}^2 \to \mathbb{R}$ are continuous functions, $k, g : \mathbb{R} \to \mathbb{R}$ are continuous functions and $a < c \leq d < b$.

We shall obtain by using the method of quasilinearization monotone iteration schemes which converge quadratically to the unique solution of the following second order four point boundary value problem

$$x'' = f(t, x), \quad t \in [a, b], \tag{1.3}$$