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ON A SYSTEM OF MAX-DIFFERENCE EQUATIONS

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Abstract. In this paper we study the periodic nature of the positive solutions of the system of difference equations:

$$y_n = \max\Big\{\frac{A_1}{z_{n-1}}, \frac{B_1}{z_{n-3}}, \frac{C_1}{z_{n-5}}\Big\}, \quad z_n = \max\Big\{\frac{A_2}{y_{n-1}}, \frac{B_2}{y_{n-3}}, \frac{C_2}{y_{n-5}}\Big\}, \; n \geq 0,$$

where $A_i, B_i, C_i,\ i \in \{1, 2, 3\}$, are positive real constants and the initial values y_i, z_i, $i \in \{-5, -4, ..., -1\}$ are positive numbers. In addition, we give conditions so that the solutions of this system be unbounded.

Keywords. System, max-difference equations, eventually periodic, unboundedness.
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1 Introduction

The fact that computer science uses mathematical models with exclusively discrete variables and not continuous, in correlation with the fact that the mathematical modeling of a real world phenomenon, very often, leads to an initial value problem of difference equations, makes difference equations one of the important sections of applied mathematics (for partial review of the theory of difference equations and their applications see [1], [3], [6], [7], [9]). Moreover, there exist many papers concerning systems of difference equations (see [4], [5], [10], [11], [13]-[15] and the references cited therein). In addition, Max operators are often used in the study of automatic control's problems and therefore, max difference equations have been attracting increasing attention in recent years (see [2], [8], [12], [18], [19], [20] and the references cited therein).

The system of difference equations we study in this paper (for system's history see [17]), was motivated by the corresponding difference equation:

$$x_n = \max\Big\{\frac{A}{x_{n-1}}, \frac{B}{x_{n-3}}, \frac{C}{x_{n-5}}\Big\}, \; n = 0, 1, ..., \tag{1.1}$$

where A, B, C are nonnegative real numbers with $A + B + C > 0$. Equation (1.1) was studied by H. Voulov in [20], who proved that every positive solution