

## The Existence of Solutions for the Nonlinear First Order Fuzzy Differential Equations with Discontinuous Conditions

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**Abstract.** In this paper, by using the directional Lipschitzian condition and Krasnoselskii's fixed point theorem, we obtain some new existence theorems of solutions for a class of initial value problems of nonlinear first order fuzzy differential equations in the metric space of normal fuzzy convex sets with distance given by maximum of the Hausdorff distance between level sets.

**Key Words:** Nonlinear first-order fuzzy differential equation; initial value problem; directional Lipschitzian condition and fixed point; existence.

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## 1 Introduction

In this paper, we study the following initial value problem of fuzzy differential equation: Find  $x : J \rightarrow E^n$  such that

$$\begin{cases} x'(t) = f(t, \lambda x(t)) + g(t, x(t)), \\ x(t_0) = x_0, \end{cases} \quad (1)$$

where  $J = [t_0, t_0 + a] \subset R = (-\infty, +\infty)$  is a compact interval,  $E^n$  is the family of all fuzzy sets  $u : R^n \rightarrow [0, 1]$ ,  $t_0 \in R$ ,  $x_0 \in E^n$ ,  $f, g : W \rightarrow E^n$  are two nonlinear functions,  $W = J \times B_b(x_0)$ ,  $a, b \in R^+ = (0, +\infty)$ ,  $B_r(\omega)$  denotes the ball which center at  $\omega$  and radius is  $r$  and  $\lambda > 0$  is a constant.

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