Dynamics of Continuous, Discrete and Impulsive Systems
Series **A:** *Mathematical Analysis* **14 (2007)** 837-850
 http://www.watam.org

NONTRIVIAL SOLUTIONS IN ABSTRACT CONES FOR HAMMERSTEIN INTEGRAL SYSTEMS

Daniel Franco[1], Gennaro Infante[2] and Donal O'Regan[3]

[1]Departamento de Matemática Aplicada
Universidad Nacional de Educación a Distancia, 28080 Madrid, Spain
dfranco@ind.uned.es
[2]Dipartimento di Matematica
Università della Calabria, 87036 Arcavacata di Rende, Cosenza, Italy
infanteg@unical.it
[3]Department of Mathematics
National University of Ireland, Galway, Ireland
donal.oregan@nuigalway.ie

Abstract. We establish new criteria for the existence of nonzero solutions of systems of integral equations of the form

$$u_i(t) = B_i\left(\int_\eta^\mu f_i(s, u_1(s), \ldots, u_n(s))\,ds\right) + \int_0^1 k_i(t,s) f_i(s, u_1(s), \ldots, u_n(s))\,ds,$$

where B_i are continuous functions, $[\eta, \mu] \subset [0, 1]$, f_i satisfy Carathéodory conditions and k_i may be discontinuous and change sign.

We apply our results to prove the existence of nontrivial solutions of some systems of differential equations with nonlinear boundary conditions.

Keywords. Cone, nonzero solution, fixed point index.
AMS subject classification: Primary 45G15, secondary 34B10, 47H10, 47H30

1 Introduction

In a recent paper Agarwal, O'Regan and Wong [1] studied the following system of integral equations

$$u_i(t) = \int_0^1 g_i(t,s) f_i(s, u_1(s), \ldots, u_n(s))\,ds, \tag{1}$$

obtaining the existence of one and multiple *constant sign* solutions. The results in the above paper are obtained by means of the Leray-Schauder alternative and Krasnosel'skii's cone compression and expansion type theorem [7]. The cone employed in [1] was a cone of *constant sign* functions.

Infante [4], when dealing with *positive* solutions of differential equations with nonlinear boundary conditions somewhat similar to the ones in [2], studied integral equations of the type

$$u(t) = B\Big(\int_\eta^\mu f(s, u(s))ds\Big) + \int_0^1 k(t,s) f(s, u(s))\,ds, \tag{2}$$