ON A HIGHER ORDER DIFFERENCE EQUATION

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Abstract. The boundedness and attractivity of the positive solutions of a quite general nonlinear difference equation is investigated.

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1 Introduction

In this paper we investigate the difference equation

\[ x_{n+1} = f(x_n, \ldots, x_{n-k})/\alpha + \beta x_{n-r}, \quad (n \geq 0) \]

where \( k \in \mathbb{N} \), \( r \in \{0, 1, \ldots, k\} \) and \( x_{-k}, \ldots, x_{-1}, x_0, \alpha, \beta \in (0, \infty) \) and \( f: [0, \infty)^{k+1} \to [0, \infty) \) is a function satisfying the following properties:

\begin{enumerate}
  \item [(H_1)] \( f(u_1, u_2, \ldots, u_{k+1}) \) is nondecreasing in each variable.
  \item [(H_2)] There are nonnegative real numbers \( A \) and \( B \) such that for each vector \((u_1, u_2, \ldots, u_{k+1})\), with \( u_i := u \) for all indices \( i \neq r + 1 \), it holds
    \[ f(u_1, u_2, \ldots, u_{r+1}, \ldots, u_{k+1}) = Au + Bu_{r+1}. \]
\end{enumerate}

We also continue our systematic treatment of difference equations, see [8-10,14-27]. For closely related results see [1-7,11-13,28,29].

Eq. (1) is a natural generalization of the difference equation

\[ y_{n+1} = \frac{\alpha y_n + \beta y_{n-r}}{A + y_n}, \quad n = 0, 1, \ldots, \]

where \( y_{-1}, y_0, A \in (0, \infty) \). The global attractivity, the boundedness character and the periodic nature of Eq. (2) was considered in [11]. A discussion of

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