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ON A HIGHER ORDER DIFFERENCE EQUATION

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Abstract. The boundedness and attractivity of the positive solutions of a quite general nonlinear difference equation is investigated.

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1 Introduction

In this paper we investigate the difference equation

$$x_{n+1} = \frac{f(x_n, \dots, x_{n-k})}{\alpha + \beta x_{n-r}}, \tag{1}$$

where $k \in \mathbf{N}$, $r \in \{0, 1, \dots, k\}$ and $x_{-k}, \dots, x_{-1}, x_0, \alpha, \beta \in (0, \infty)$ and $f : [0, \infty)^{k+1} \to [0, \infty)$ is a function satisfying the following properties:

(H_1) $f(u_1, u_2, \dots, u_{k+1})$ is nondecreasing in each variable.

(H_2) There are nonnegative real numbers A and B such that for each vector $(u_1, u_2, \dots, u_{k+1})$, with $u_i =: u$ for all indices $i \neq r+1$, it holds

$$f(u_1, u_2, \dots, u_{r+1}, \dots, u_{k+1}) = Au + Bu_{r+1}.$$

We also continue our systematic treatment of difference equations, see [8-10,14-27]. For closely related results see [1-7,11-13,28,29].

Eq. (1) is a natural generalization of the difference equation

$$y_{n+1} = \frac{\alpha y_n + \beta y_{n-1}}{A + y_n}, \quad n = 0, 1, \dots, \tag{2}$$

where $y_{-1}, y_0, A \in (0, \infty)$. The global attractivity, the boundedness character and the periodic nature of Eq. (2) was considered in [11]. A discussion of

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