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OSCILLATION OF SECOND ORDER NONLINEAR NEUTRAL EQUATIONS WITH DISTRIBUTED DEVIATING ARGUMENTS

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Abstract. Oscillation criteria are established for the second order nonlinear neutral delay differential equation with distributed deviating argument

$$(r(t)\psi(x(t))|Z'(t)|^{\alpha-1}Z'(t))' + \int_a^b q(t,\xi)\, f[x(g(t,\xi))]d\sigma(\xi) = 0, \quad t \geq t_0,$$

where $Z(t) = x(t) + p(t)x(t-\tau)$ and $\alpha > 0$. These results are extensions of the integral averaging techniques due to Kamenev, Philos and Wong, and improve some known oscillation criteria in the literature.

Keywords. Oscillation; Neutral delay differential equation; nonlinear; Second order; Distributed deviating argument.

AMS (MOS) subject classification: 34C10, 34K11; 34K40.

1 Introduction

In this paper we are concerned with the oscillation problem of the second order nonlinear neutral delay differential equation with distributed deviating argument

$$(r(t)\psi(x(t))|Z'(t)|^{\alpha-1}Z'(t))' + \int_a^b q(t,\xi)\, f[x(g(t,\xi))]d\sigma(\xi) = 0, \quad t \geq t_0, \tag{1.1}$$

where $Z(t) = x(t) + p(t)x(t-\tau)$, $\tau \geq 0$, $\alpha > 0$, and the following conditions are assumed to hold in this paper:

(A1) $r, p \in \mathbf{C}(I, \mathbb{R})$, $0 \leq p(t) \leq 1$, and $r(t) > 0$ for $t \in I$, $\int^\infty (r(s))^{-1/\alpha} ds = \infty$, $I = [t_0, \infty)$;

(A2) $\psi \in \mathbf{C}^1(\mathbb{R}, \mathbb{R})$, $\psi(x) > 0$ for $x \neq 0$;

(A3) $f \in \mathbf{C}(\mathbb{R}, \mathbb{R})$, $xf(x) > 0$ for $x \neq 0$;

(A4) $q \in \mathbf{C}(I \times [a,b], [0,\infty))$ and $q(t,\xi)$ is not eventually zero on any half-linear $[t_u, \infty) \times [a,b]$, $t_u \geq t_0$;