

OSCILLATORY AND ASYMPTOTIC BEHAVIOR OF ODD ORDER NEUTRAL DIFFERENTIAL EQUATIONS

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Abstract. In this paper we establish sufficient conditions for the existence of oscillatory solutions of odd order neutral differential equations with forcing term

$$[x(t) - px(t - \tau)]^{(n)} + \sum_{j=1}^m q_j(t)x(t - \sigma_j) = h(t).$$

These conditions are based on the oscillation of first order delay differential equation. The asymptotic properties of their nonoscillatory solutions are also investigated.

Keywords. Neutral equations, Asymptotic behavior, Oscillations, Nonlinear, Delay differential equations.

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1. Introduction

In the present paper we consider the oscillatory behavior of neutral functional differential equations of the form

$$[x(t) - px(t - \tau)]^{(n)} + \sum_{j=1}^m q_j(t)x(t - \sigma_j) = h(t) \quad (1)$$

and

$$[x(t) - p(t)x(t - \tau)]^{(n)} + q(t)f(x(t - \sigma)) = h(t), \quad (2)$$

where $q_j \in C([t_0, \infty), [0, \infty))$, $p, q \in C([t_0, \infty), (0, \infty))$, $f, h \in C(\mathbb{R}, \mathbb{R})$, $\sigma_j > 0$ for $j = 1, 2, \dots, m$, $0 < p < 1$, $\tau, \sigma > 0$, $xf(x) > 0$, $x \neq 0$ and n odd.

The oscillation criteria of Eq. (1) and (2) has been investigated by several authors Das [3], Gopalsamy et al., [4] Ladas et al. [6] and Shen [8]. However, most of these papers involve homogeneous equations.

In this paper we study the oscillatory behavior of Eq. (1) and (2) involving forcing term and relating to first order differential equation. Sufficient conditions for oscillatory and nonoscillatory behavior are presented with an example.