Dynamics of Continuous, Discrete and Impulsive Systems
Series **B:** *Applications & Algorithms* **14 (2007)** 625-640
 http://www.watam.org

THE LURIE CONTROL SATISFIES A LIÉNARD EQUATION

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Abstract. We transform the Lurie indirect control equations into Liénard type integro-differential equations. Depending on the number of zero eigenvalues of the matrix in the Lurie equations, we obtain a pure Liénard equation, a Liénard equation with memory and an exponentially decaying forcing function, and a Liénard-Volterra-Levin equation. Using the good Liapunov functions known for these type of equations, we prove stability results for the Liénard equations and consequently for the indirect control Lurie problem. The results are then extended to the delay form of the Lurie problem.
Keywords. Lurie, Liapunov function, Liénard equation, Control.
AMS (MOS) subject classification: 34A34, 34D05, 34K20, 93D05.

1 Introduction

The problem of Lurie began in 1951 [10] and has attracted much interest to the present time, ever changing to encompass more sophisticated systems, but remaining basically the same. A search of the topic "Lurie" in the online Mathematical Reviews will net 85 papers, which are but a fraction of the total literature on the problem, as can be seen by then searching the publications of the authors listed in that first 85. The last 55 of that 85 paper set have appeared since 1995; thus, there is a great resurgence of interest in the subject.

The basic problem is how to ensure the stability of a linearized plant equation using a scalar control that is a non-linear function of the error.

We assume that the plant equation is given by

$$x' = Ax$$

where A is a $d \times d$ real constant matrix having m zero characteristic roots and n roots with negative real parts.

A control is added to the system and a transformation is made resulting