Dynamics of Continuous, Discrete and Impulsive Systems
Series **A:** *Mathematical Analysis* **14 (2007)** 739-753
 http://www.watam.org

THE METHOD OF UPPER AND LOWER SOLUTIONS FOR NTH ORDER NONLINEAR IMPULSIVE DIFFERENTIAL INCLUSIONS

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Abstract. In this paper, using the method of upper and lower solutions and the fixed point theorem due to Martelli, we present sufficient conditions for the existence of solutions of initial value problems for nth order nonlinear impulsive differential inclusions on an infinite interval with infinite number of impulsive times under Caratheodory conditions.
Keywords. Impulsive differential inclusion; initial value problem; Multivalued map; Fixed point.

1 Introduction

Let $\mathbb{R}$ be the real line and let $J_a = [0, a]$ $(0 < a < \infty)$ denote a closed and bounded interval in $\mathbb{R}$ and $J = [0, \infty)$. Consider the initial value problem (IVP) for the nth order nonlinear impulsive differential inclusion

$$\begin{cases} u^{(n)} \in F(t, \overline{u}), & \forall t \in J, t \neq t_k (k = 1, 2, \cdots), \\ \Delta u^{(i)}|_{t=t_k} = I_{ik}(\overline{u}(t_k)) & (i = 0, 1, \cdots, n-1, k = 1, 2, \cdots), \\ u^{(i)}(0) = u_i & (i = 0, 1, \cdots, n-1), \end{cases} \tag{1}$$

where $F : J \times \mathbb{R}^n \to 2^{\mathbb{R}}$, $0 < t_1 < t_2 < \cdots < t_k < \cdots, t_k \to \infty, I_{ik} \in C[\mathbb{R}^n, \mathbb{R}](i = 0, 1, \ldots, n-1; k = 1, 2, \cdots), \overline{u} = (u, u', \cdots, u^{(n-1)})$, $\overline{u}(s) = (u(s), u'(s), \cdots, u^{(n-1)}(s))$, $u_i \in \mathbb{R}$ $(i = 0, 1, \cdots, n-1)$. $\mathbb{R}^n$ is the n dimensional Euclidean space. $\Delta u^{(i)}|_{t=t_k}$ denotes the jump of $u^{(i)}$ at $t = t_k$, i.e.,

$$\Delta u^{(i)}|_{t=t_k} = u^{(i)}(t_k^+) - u^{(i)}(t_k^-),$$

where $u^{(i)}(t_k^+)$ and $u^{(i)}(t_k^-)$ represent the right and left limits of $u^{(i)}(t)$ at $t = t_k$, respectively.

The theory of impulsive differential equations and inclusions has become important in some mathematical models of real processes and phenomena studied in physics, chemical technology, population dynamics, biotechnology and economics, and this theory has been emerging as an important area of investigation (see [9,14]). But most of the works in this area discussed the first- and second- order equations (see [3, 4, 9, 12, 13]), and the theory of