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A PARTIAL DIFFERENTIAL EQUATION WITH DELAYED DIFFUSION

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Abstract. We consider a simple diffusion equation with a delayed Laplacian operator to model the time required for spatial movement. Such an equation defines a semiflows on a Frechét space and the associated infinitesimal generator has a quite interesting spectral property so that the associated unstable and stable subspaces are both infinitely dimensional, and there is a sequence of eigenvalues of the generator that approaches the imaginary axis.

Keywords. delay diffusion, separation of invariant manifolds, semigroup on Frechét spaces.

AMS (MOS) subject classification: Primary: 35B40, 35G15, 34K25, 35P15, 35R10.

1 Introduction

Motivated by a simple looking linear parabolic equation with delay in the Laplacian operator, we consider here semiflows on general Frechét spaces. The need for such a framework instead of C_0-semigroups on Banach spaces is due to the fact that solutions of the aforementioned equation become less and less smooth (with respect to space) as time increases, and hence the space of smooth initial conditions is required in order for the abstract Cauchy initial value problem to be well-posed.

Here we present a case study of the simplest possible diffusion equation where diffusion occurs with a delay, and we show that such an equation generates a semigroup of bounded operators in a carefully chosen Frechét space whose metric is induced by a family of semi-norms. The generator of such a semiflow can be calculated, and we show its spectrum contains a sequence of points with unbounded positive real part, and hence both stable subspaces and unstable subspaces of the semiflow must be of infinite dimensions. Our analysis shows the lack of exponential dichtomy as there is a sequence of eigenvalues of the generator that approach the imaginary axis.