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R_δ SOLUTION SETS OF INTEGRAL INCLUSIONS ON A NONCOMPACT INTERVAL VIA MAXIMAL SOLUTIONS

Donal O'Regan[1] and Xinzhi Liu[2]

[1]Department of Mathematics, National University of Ireland, Galway, Ireland
[2]Department Of Applied Mathematics, University of Waterloo, Waterloo, Ontario, Canada N2L 3G1

Abstract. In this paper we show that the solution set of certain Volterra integral inclusions defined on half open intervals is an R_δ set.

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1 Introduction

This paper discusses the solution set of the Volterra integral inclusion

$$y(t) \in \int_0^t k(t,s)\, F(s, y(s))\, ds \quad \text{for} \quad t \in [0,T); \tag{1.1}$$

here $0 < T \leq \infty$ is fixed. Let $t_n \uparrow T$. We show that the solution set of (1.1) is an R_δ set if F is bounded by a L^1_{loc}-Carathéodory function g (with g nondecreasing in x for a.e. $t \in [0,T)$) and if

$$\begin{cases} \text{for each } n \in N = \{1,2,...\}, \text{ the problem} \\ \begin{cases} v'(t) = \left(\sup_{t\in[0,t_n]} k(t)\right) g(t, v(t)) \text{ a.e. } t \in [0,t_n] \\ v(0) = 0 \end{cases} \\ \text{has a maximal solution } r_n(t) \text{ on } [0,t_n] \end{cases}$$

and

$$\begin{cases} \text{there exists a } \psi \in C[0,T) \text{ such that for each} \\ n \in N \text{ we have } r_n(t) \leq \psi(t) \text{ for } t \in [0,t_n] \end{cases}$$

hold. Our theory extends and complements results in [1, 3, 4]. For example Theorem 1.2.4 of [3] follows immediately from Theorem 3.4. Some of the results in section 3 were motivated by ideas presented by Andres, Gabor and Gorniewicz [4], and Agarwal and O'Regan [3].