TWO-STEP MACCORMACK METHOD FOR
STATISTICAL MOMENTS OF A STOCHASTIC
BURGER’S EQUATION

Hongjoong Kim
Department of Mathematics, Korea University,
1, 5-Ga, Anam-Dong, Sungbuk-Ku, Seoul, 136-701, Korea

Abstract. The two-step MacCormack scheme has been modified to solve a stochastic
Burger’s equation driven by a random force with a random initial condition. Statistical
moments of a solution are expressed by Hermite-Fourier coefficients so that the stochas-
tic equation is transformed into a deterministic propagator system. The resultant system
needs to be solved only once and computational loads are reduced accordingly. The nu-
merical stability, accuracy and efficiency of the scheme have been analyzed and compared
with the Monte Carlo method and the Lax-Wendroff scheme. The modified MacCormack
scheme shows less diffusion near discontinuities than the Lax-Wendroff scheme. While
maintaining the same accuracy, the MacCormack scheme improves numerical efficiency
over the Lax-Wendroff scheme in the ratio of \((N + 11/6)\) when the length is \(N\). Compared
to the Monte Carlo method, the scheme saves more than 98% of CPU time and removes
dependence upon a random number generator.

Keywords. MacCormack scheme, Stochastic differential equation, Burger’s equation,
Lax-Wendroff scheme, Monte Carlo method

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1 Introduction

It is common to view uncertainty in mathematical models such as Euler
equations for gas dynamics, the Buckley-Leverett equation for multi-phase
flow in porous media [9, 10, 11] or the Navier-Stokes equation [16, 23]. This
may result from insufficient information such as imprecise coefficient or input
values [28]. Limited understanding of underlying phenomena may cause such
a noise [13]. Observational errors due to inaccuracies in the measurements
may not be removed and lead to a filtering problem [28, 32]. Black-Scholes
model for option pricing in finance [18] is another example for uncertainties.
In order to estimate these effects, uncertainties are mathematically modeled
by random fields. Then, the governing equation is represented in the form
of a stochastic partial differential equation (SPDE). Since the solution of
this type of equation is random, one particular solution corresponding to
a specific realization is not of concern. Instead, it is important to know
statistical properties of the solution such as its mean or variance.