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ON SOME CLASSES OF LIMIT CYCLES OF PLANAR DYNAMICAL SYSTEMS

A. Grin[1] and K.R. Schneider[2]

[1] Grodno State University
Ozheshko Street 22
230023 Grodno, Belarus
[2] Weierstrass Institute for Applied
Analysis and Stochastics
Mohrenstraße 39, 10117 Berlin, Germany

Abstract. We consider two-dimensional smooth vector fields $dx/dt = P(x, y)$, $dy/dt = Q(x, y)$ and estimate the maximal number of limit cycles with special properties which are defined by means of generalized Dulac and Cherkas functions. In case that P and Q are polynomials we present results about the weakened 16-th problem of D.Hilbert.
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1 Introduction

We consider two-dimensional systems of autonomous differential equations

$$\frac{dx}{dt} = P(x, y), \; \frac{dy}{dt} = Q(x, y) \tag{1}$$

in some region $D \subset R^2$. Throughout the paper we assume that P and Q are continuously differentiable in D, that is, $P, Q \in C^1(D)$. If D is bounded and such that the boundary of D has no contact with the trajectories of system (1), then according to the skeleton method of A.A. Andronov and E.A. Leontovich [1] the topological structure of the trajectories of (1) in D is determined by the singular trajectories of system (1), that is, by the equilibria, the separatrices and the limit cycles of (1) in D.
There are sufficiently effective methods to study equilibria and separatrices, there are also methods to prove that (1) has no limit cycle or at least one limit cycle in D (see, e.g. [4, 19, 25, 27]), but there is no general method to localize all limit cycles and to prove the existence or absence of multiple limit cycles. Therefore, the problem to estimate the number of limit cycles of general systems (1) is an open problem, even in the case of polynomial systems

$$\frac{dx}{dt} = \sum_{i+j=0}^{n} a_{ij} x^i y^j, \; \frac{dy}{dt} = \sum_{i+j=0}^{n} b_{ij} x^i y^j \tag{2}$$