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THE EIGENVALUES AND EXISTENCE OF SOLUTIONS OF BVPS FOR FOURTH ORDER DIFFERENCE EQUATIONS

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Abstract. This paper investigates the existence of the eigenvalue pair of linear boundary value problems for fourth order difference equations. By employing the upper and lower solution method, we establish the existence of solutions for nonlinear boundary value problems of fourth order difference equations under a nonresonance condition involving a two-parameter eigenvalue problems.

Keywords. Eigenvalues, upper and lower solutions, difference equations.
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1 Introduction

The fourth order linear two-parameter eigenvalue problems of differential equation

$$\begin{cases} y^{(4)}(t) = \alpha y(t) - \beta y''(t), \quad 0 < t < 1, \\ y(0) = y(1) = y''(0) = y''(1) = 0, \end{cases} \tag{1.1}$$

have been investigated by M. A. Del Pino and R. F. Manàsevich in [10]. They called the pair (α, β) as an eigenvalue pair when (1.1) has a nontrivial solution for a pair parameter (α, β). Moreover they gave the necessary and sufficient conditions for a pair parameter (α, β) is an eigenvalue pair for problem (1.1). On the other hand, the scholars [cf. 1, 6-8, 11, 15-19] also investigated the nonlinear boundary value problems of differential equation

$$\begin{cases} y^{(4)}(t) = f(t, y(t), y''(t)), \quad 0 < t < 1, \\ y(0) = y(1) = y''(0) = y''(1) = 0, \end{cases} \tag{1.2}$$

Under the nonresonance conditions involving a two-parameter linear eigenvalue problems, they established the existence of solutions of BVPs (1.2) when the nonlinear term f satisfies Lipschitz condition or has the monotone properties [cf. 6, 15, 16]. Especially, by the upper and lower solution method, many scholars have investigated the existence of solutions to (1.2) [cf. 6, 8, 11, 16]. For instance, in a recent paper [18], Y. Wang investigated the existence and uniqueness of solutions of boundary value problems (1.2) by the