

CAUCHY PROBLEM FOR AN ENSKOG EQUATION WITH ATTRACTIVE TAIL AND FOKKER-PLANCK SCATTERING

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Abstract. To model a dense gas, a weak attractive long range potential is added to the Enskog collision operator, along with diffusions to represent thermal background interaction and dynamical friction forces. The resulting Fokker-Planck Vlasov-Enskog equation is coupled to a Poisson equation. Stability of solution sets is studied, and the existence of global-in-time solutions to the renormalized equation is proved for arbitrary L^1 initial conditions with finite mass, energy and entropy.

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1 Introduction

Construction and analysis of kinetic equations for dense gases is one of the most important continuing problems in the kinetic theory of classical systems. The Boltzmann equation, first written in its modern form in 1876, describes the evolution of the differential density function (single particle distribution function) $f(\vec{r}, \vec{v}, t)$ of a gas. It is the dominant equation of kinetic theory, and its collision kernel allows the Boltzmann equation to model a wide variety of intermolecular potentials. However, the equation is known to be valid only in the dilute-gas regime, in fact describing a gas of point particles and yielding transport coefficients of an ideal gas.

To model moderately dense gases, Enskog in 1921 proposed a Boltzmann-like kinetic equation, representing molecules with non-zero diameter a . The Enskog equation, as revised in the 1970's in order to obtain correct hydrodynamics, describes a non-ideal fluid with transport coefficients within 10% of those of realistic gases up to one-half close packing density. A limitation, however, in its usefulness is that, unlike the Boltzmann equation, no molecular interaction is modelled beyond the collision of hard spheres.

A strategy to rectify this is the addition of a smooth attractive tail to the hard repulsive core, thereby more closely approximating a van der Waals molecular interaction. The potential must be introduced at the Liouville