Dynamics of Continuous, Discrete and Impulsive Systems
Series **A:** *Mathematical Analysis* **14 (2007)** 615-630
 http://www.watam.org

COMPUTATION OF THE BIFURCATING PERIODIC SOLUTIONS FOR REACTION-DIFFUSION EQUATIONS

Kouichi Murakami

Department of Mathematical Sciences, The University of Tokushima
Tokushima 770-8502, JAPAN

Dedicated to Professor Tadayuki Hara on the occasion of his 60th birthday

Abstract. We propose a simplified formula to approximate the center manifold to analyze the Hopf bifurcation for reaction-diffusion equations. Also we give the asymptotic approximation of the bifurcating periodic solution. Using our result, we compute the spatially nonhomogeneous time-periodic solution of the Brusselator.
Keywords. reaction-diffusion equation, Hopf bifurcation, periodic solution

1 Introduction

The purpose of this paper is to compute the spatially nonhomogeneous time-periodic solution for reaction-diffusion equations. In their book, Hassard and others present the procedure to analyze the Hopf bifurcation and give the explicit expression of the bifurcating periodic solution for several types of equations ([1]). Although their principle is straightforward, the procedure requires long complicated calculations, especially in case that the equations are in higher dimension and include quadratic nonlinearities.

In this paper, we propose a simplified formula to approximate the center manifold to analyze the Hopf bifurcation. Also we give the asymptotic approximation of the bifurcating periodic solution. Our expression of the periodic solution is better than the above mentioned results. We already have the same result for other types of equations ([3, 4]). This paper shows that the same result holds for reaction-diffusion equations, too.

In section 2, we present our result to analyze the Hopf bifurcation. Also we give the asymptotic approximation of the bifurcating periodic solution. In section 3, we apply our result to the Brusselator which is studied in [1, 2], and illustrate our result by numerical examples.

2 Hopf bifurcation analysis

Consider the reaction-diffusion equation

$$u_t = D\,u_{xx} + A_\mu u + F_\mu(u) \tag{2.1}$$