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TIME-OPTIMAL CONTROL OF A 3-LEVEL QUANTUM SYSTEM AND ITS GENERALIZATION

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Abstract. We solve the problem of steering a three-level quantum system from one eigenstate to another in minimum time and study its possible extension to the time-optimal control problem for a general n-level quantum system. For the three-level system we find all optimal controls by finding two types of symmetry in the problems: $\mathbb{Z}_2 \times S_3$ discrete symmetry and S^1 continuous symmetry, and exploiting them to solve the problem through discrete reduction and symplectic reduction. We then study the geometry, in the same framework, which occurs in the time-optimal control of a general n-level quantum system.

Keywords. Time-optimal control, quantum systems, Hamiltonian mechanics, symmetry, symplectic reduction.

1 Problem Statement

In this paper we study the time-optimal control problem for the following 3-level system:

$$\begin{cases} \dot{x}_1 = -\omega_3 x_2 \\ \dot{x}_2 = \omega_3 x_1 - \omega_1 x_3 \\ \dot{x}_3 = \omega_1 x_2 \end{cases} \tag{1}$$

with the initial and final conditions

$$\mathbf{x}(0) = (1,0,0), \quad \mathbf{x}(T_{\min}) = (0,0,1) \tag{2}$$

and the control constraints

$$|\omega_1| \le 1, \quad |\omega_3| \le 1. \tag{3}$$

We show that there are exactly two optimal control laws which are

$$(\omega_1, \omega_3) = \pm(1,1)$$