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SINGULAR HIGHER-ORDER SEMIPOSITONE NONLINEAR EIGENVALUE PROBLEMS

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Abstract. In this paper, we consider the existence of positive solutions to the following singular higher-order semipositone eigenvalue problem (HSEP):

$$\begin{cases} (-1)^{(n-k)}u^{(n)}(t) = \lambda\Big[f(t,u(t)) + q(t)\Big], & 0 < t < 1, \\ u^{(i)}(0) = 0, & 0 \le i \le k-1, \\ u^{(i)}(1) = 0, & 0 \le i \le n-k-1, \end{cases}$$

where $n \ge 2, 1 \le k \le n-1$, and $\lambda > 0$ is a parameter. The functions f and q may have singularity at $t = 0$ and (or) 1, and furthermore, the nonlinear function may change sign for $0 < t < 1$. Without making any monotone-type assumptions, we obtain the positive solution of the problem for λ lying in some interval, based on the Krasnaselskii's fixed-point theorem in a cone.

Keywords. Positive solution; Semipositone; Singular eigenvalue problem; Fixed point; Cone.

AMS (MOS) subject classification: 34B15, 34B25.

1 Introduction

In this paper, we study the existence of positive solutions for the following singular higher-order semipositone eigenvalue problem (HSEP):

$$\begin{cases} (-1)^{(n-k)}u^{(n)}(t) = \lambda\Big[f(t,u(t)) + q(t)\Big], & 0 < t < 1, \\ u^{(i)}(0) = 0, & 0 \le i \le k-1, \\ u^{(i)}(1) = 0, & 0 \le i \le n-k-1, \end{cases} \tag{1.1}$$

where $n \ge 2, 1 \le k \le n-1$, $\lambda > 0$, $f : (0,1) \times [0,+\infty) \to [0,+\infty)$ and $q(t) : (0,1) \to (-\infty,+\infty)$ are continuous and may have singularity at $t = 0$ and (or) 1. Furthermore, the nonlinear function is allowed to change sign.