Dynamics of Continuous, Discrete and Impulsive Systems
Series **A:** *Mathematical Analysis* **14 (2007)** 557-564
 http://www.watam.org

EXISTENCE OF SOLUTIONS FOR SINGULAR DIFFERENTIAL INCLUSIONS

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Abstract. We extend a result of Cherpion and De Coster concerning existence of solutions for singular differential equations to the setting of differential inclusions. Lower and upper solutions are employed and an example is given to illustrate the use of the theorem.
Keywords. Existence of solutions, singular, differential inclusion, lower solution, upper solution.
AMS (MOS) subject classification: 34A60, 34A12.

We consider the problem of extending the result of Cherpion and De Coster [11] for existence of solutions for singular differential equations to the setting of differential inclusions. Existence of solutions for singular differential equations has been studied extensively, including a great deal of work by Agarwal, O'Regan and co-authors. As of now, their work has culminated in the result in [7]. Recently, the hypotheses in the result in [11] have been generalized [12]. (We note that extending the results of our paper to a differential inclusion version of [12] would require a number of major changes, including for example, a new Lemma 1, a research project in itself.) A sampling of other recent works which have considered singular first-order initial value problems include [1], [2], [3], [4], [6], [10], [13], [14]. A good general resource is [5]. However, differential inclusions with singularities in the state variable do not seem to have been considered to this point.

We consider the problem of finding solutions of the problem

$$\begin{aligned} u'(t) &\in F(t,u(t)),\ t\in\ [0,T] \qquad (1)\\ u(0) &= 0 \end{aligned}$$

where $F(t,x):(0,T]\times(0,\infty)\to 2^{\mathbf{R}}$ might have singularities at $t=0$ and/or $x=0$. Our approach follows that of [11] with modifications to handle the set-valued right-hand side. We define solution as follows.

Definition 1: $u:[0,T]\to\mathbf{R}$ is a solution of (1) if it satisfies:

1) u is absolutely continuous on compact subintervals of $(0,T]$,
2) u is continuous on $[0,T]$,