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THE EXISTENCE OF SOLUTIONS FOR NONLINEAR TWO-POINT BOUNDARY PROBLEMS WITH PARAMETER UNDER AMBROSETTI-PRODI CONDITION

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Abstract. In this paper, we discuss the number of solutions of certain ordinary differential equations with parameter under Ambrosetti-Prodi condition. Two important results are provided. One is to present the range of parameter, in which there is no solution for original problem. The other one is to investigate the existence of multiple solutions by strict upper and lower solution and coincidence topological degree, and provide the corresponding range of parameter.
Keywords. Nonlinear boundary problem, Strict upper and lower solution, Multiple solution, Equicontinuous, Ambrosetti-Prodi condition, Coincidence topological degree.

1 Introduction

It is well known that many practical problems in engineering, mathematical finance and some other scientific subjects are nonlinear boundary value problems. So, it is very useful and important to analyze the property of solutions for these kinds of problems.

In the past thirty years, many papers (see [4], [3], [7], [10], [11], [13], [12], [14], [15], etc) discussed nonlinear ordinary differential systems under generalized Ambrosetti-Prodi conditions. In [4], a nonlinear two-point boundary value problem as follows is considered:

Find $u : [0,\pi] \to R$ of class C^2, such that

$$u'' + u + f(x,u) = s\sqrt{2/\pi}\sin x, \qquad u(0) = u(\pi) = \gamma, \tag{1}$$

where $f : [0,\pi] \times R \to R$ is continuous, $s \in R$, $\gamma \in R$. Moreover, in [4], under the assumption that

$$\lim_{|u|\to\infty} f(x,u) = +\infty$$

uniformly for $x \in [0,\pi]$, the authors proved an important result that: there exist s_0, $s_0^+ \in R$ ($s_0 \le s_0^+$) such that (1) with $\gamma = 0$ has no solution for