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COMPACTNESS OF SOLUTIONS TO SECOND ORDER STOCHASTIC DYNAMICAL SYSTEMS

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Abstract. The purpose of the paper is to study compactness properties of the solution set for a second order stochastic inclusion driven by two semimartingales. The connection between such an inclusion and the martingale problem is established. This property is crucial for the problem. The results extend those known for first order stochastic equations and inclusions.

Keywords. semimartingale, stochastic set-valued integral, stochastic inclusion, martingale problem.

AMS (MOS) subject classification: 93B05; 93C30; 93E03; 60H20.

1 Introduction

Deterministic control problems can be described in connection with the set-valued analysis and differential inclusions theory ([5, 18]). Similarly, the theory of stochastic inclusions appears as a theoretical description of stochastic control problems (e.g. [1, 2, 3, 7, 8, 19, 20, 21, 22, 24, 25, 28] and references therein). On the other hand, nowadays there is a great interest in investigating second order differential inclusions in connection with controllability of deterministic dynamical systems described by second-order differential equations [9, 17, 23, 30, 31]. Therefore, it seems quite natural to take interest in the second order stochastic inclusion. In [26] the authors studied the existence of solutions for such a type inclusion driven by general semimartingales. In the present work, we continue the study. By a similar approach to the problem as that used for a first order and single valued case in ([14, 15, 16, 34]), we show the equivalence between the existence of solutions to a second order stochastic inclusion and the martingale problem for such an inclusion. As a consequence we obtain the compactness of the set of all solutions of such inclusion. Compactness often appears as a desirable property in many problems in economy, insurance, finance, and so forth (see e.g.: [27]).

The paper is organized as follows. Section 2 contains preliminaries and auxiliary facts connected with set-valued mappings and semimartingales. In Section 3, we introduce the notion of a driving system and we prove the existence of solutions to second order stochastic inclusions (Theorem 3.7 and