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# THE EXISTENCE AND UNIQUENESS OF LIMIT CYCLES FOR A CLASS OF PLANAR DIFFERENTIAL SYSTEMS

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**Abstract:** In this paper, we study a class of planar differential systems $\dot{x} = y(1 + \sum_{i=1}^{\infty} b_i x^i)$, $\dot{y} = -x + \delta y + a_1 x^2 + a_2 xy + a_3 x^2 y + \lambda x^4 e^x y$. By using the Hopf bifurcation theory, some sufficient conditions for the existence and stability of limit cycles of such systems are obtained. Furthermore, by applying the L.A.Cherkas and L.I.Zheilevych's theorem about uniqueness, some sufficient conditions for the uniqueness of limit cycles of such systems are established.
**Keywords:** Planar differential systems; Limit cycle; Existence;Uniqueness.
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# 1 Introduction

In the qualitative theory of differential equations, A classical problem of real planar differential equations is the determination of limit cycles. Limit cycles of planar vector fields were defined by Poincaré in the 1880s [1]. A limit cycle is an isolated periodic orbit,that is, a periodic orbit such that there exists an annular neighborhood without any other closed orbit. Poincaré showed that if the two-dimensional autonomous systems $\dot{x} = P(x, y)$, $\dot{y} = Q(x, y)$ has a weak focus (i.e.a critical point which is a center for the corresponding variation equation ), then limit cycles can be made to appear in the neighborhood by varying slightly the right hand sides, see[1-2]. Then, in the famous list of 23 mathematical problems stated in 1900,in the International Congress of Mathematics in Paris, D.Hibbert asked in the second part of 16th problem for an upper bound for the number of limit cycles for the equation $\dot{x} = P(x, y)$, $\dot{y} = Q(x, y)$ ($P_n(x, y)$,$Q_n(x, y)$ are polynomials of real variables $x, y$ with real coefficients and degree not higher than n) and their relative positions. Of the results obtained in the last century, we mention the