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A UNIQUENESS RESULT FOR STEADY SYMMETRIC WATER WAVES WITH AFFINE VORTICITY

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Abstract. We prove uniqueness for steady symmetric water waves with affine vorticity. The result holds with or without surface tension and gravity.
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1 Introduction

The study of water waves propagating on a rotational current has seen an increasing interest in the past decade. Within this context, the existence of steady water waves with vorticity has been proved using bifurcation theory [5, 11, 18, 19, 20] and a variational formulation for such waves is available [4]. Under certain assumptions on the vorticity, the papers [7, 13] offer sufficient conditions for uniqueness in the case of steady water waves on infinite and finite depth, respectively: given the surface profile and the vorticity distribution, the fluid motion is completely determined. This note is concerned with waves propagating at the surface of water over a flat bed. It asserts that, for uniqueness of symmetric waves with affine vorticity, the auxilliary assumption on the vorticity in [7, 13] can be dropped. In particular, for such waves the bound presented in [13] is unnecessary.

Symmetric waves seem to be common. In the case of a general vorticity, all solutions found so far [5, 11, 18, 19, 20] are symmetric. We also know that if the surface profile is monotone, then reasonable assumptions on the vorticity force the wave to be symmetric [2, 3], the surface profile being either flat or strictly increasing from trough to crest [8].

We would like to point out that this note is independent of the exact values of the surface tension and gravity constants. Hence uniqueness includes both capillary, capillary-gravity, and gravity waves (for a general reference on the problem considered in this paper, see e.g. [5, Section 1]). The proof is based on classic results on the Dirichlet eigenproblem for the Laplacian [12, 15], and repeatedly utilizes the Bernoulli surface condition for water waves. It also makes some use of the Boundary Point Lemma for maximum principles [9].