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ON SIMPLE MODELS OF THE NORTH ATLANTIC THERMOHALINE CIRCULATION WITH SWITCHING

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Abstract. It is commonly (though not unanimously) believed that the northward heat transport in the North Atlantic Ocean is controlled by the slow (millenial time scale) circulation due to density differences, often called the thermohaline circulation. It is believed that the thermohaline circulation underwent massive changes during the deglaciation process, between 20 and 8 thousand years ago. Perhaps more surprisingly, analysis of ice and sediment cores indicates that fluctuations during the glacial climate were nearly as large. While state–of–the–art–coupled climate models are in reasonable agreement with proxy data from sediment and ice cores, a long tradition of conceptual models that capture the 'essence' of a given physical process exists, and often provides the prism through which climate simulations are interpreted. In this article we examine a recent three equation model of the thermohaline circulation that includes switching. We employ a two-dimensional linear model with switching to illustrate how the introduction of switching implies that a limit cycle coexists with a stable equilibrium point. Subsequently we provide approximate analytical and exact numerical estimates for the basin of attraction of the fixed point. Finally we discuss the implications of our results for models of the North Atlantic thermohaline circulation.

Keywords. thermohaline circulation, glacial climates, ordinary differential equations with switching, limit cycles, asymptotic stability

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1 Introduction

From the ebb and flow of sea level that accompanies the 100,000 year ice age cycle, through the multi-millenial redistribution of heat from equator to pole driven by small changes in salinity and temperature, to the meanders and eddies of the wind-driven currents like the Gulf Stream that change from season to season, the circulation of the world's oceans varies on a range of space and time scales that is truly awe-inspiring. The mathematical models for these motions, while still rudimentary when compared against the natural world, also span a wide variety of methodologies and end goals. Modern climate models typically include several sets of nonlinear partial differential equations, each representing some component of the climate. The ocean is