

EXISTENCE OF SOLUTIONS FOR SINGULAR IMPULSIVE BOUNDARY VALUE PROBLEMS ON THE HALF-LINE ¹

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Abstract. This paper discusses the existence of solutions for singular impulsive boundary value problems on the half-line. By the method of upper and lower solutions, some necessary and sufficient conditions for the existence of solutions are obtained.

Keywords. Boundary value problems, singularity, impulses, lower and upper solutions, existence.

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1 Introduction

In this paper, we consider the impulsive differential problems

$$\begin{cases} x'' = f(t, x, x'), & t > 0, t \neq t_1, \\ \Delta x|_{t=t_1} = I(x(t_1)), \\ \Delta x'|_{t=t_1} = 0 \end{cases} \quad (1.1)$$

with the boundary value conditions

$$x(0) = r, \quad x(\infty) = \text{const}, \quad (1.2)$$

$$x(0) = r, \quad x'(\infty) = l, \quad (1.3)$$

where $\Delta x|_{t=t_1} = x(t_1 + 0) - x(t_1 - 0) = x(t_1+) - x(t_1)$, $\Delta x'|_{t=t_1} = x'(t_1 + 0) - x'(t_1 - 0)$, $f \in C(J \times \Omega \times R, R)$, $r \in \Omega$, J, Ω are nonempty open intervals in R , and f may be singular at $t = 0$ or $x = 0$, $I(x) \in C(R, R)$, and I is nondecreasing in x . In (1.2), the limit $x(\infty)$ is a constant not previously given, but in (1.3) l is a given constant.

This paper we always assume the following hypotheses:

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