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GLOBAL EXISTENCE, EXPONENTIAL DECAY, AND BLOW-UP IN ONE-DIMENSIONAL QUASILINEAR HYPERBOLIC SYSTEMS : A UNIFIED APPROACH

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Abstract. Classical solutions for nonlinear hyperbolic systems may decay or blow up depending on the damping factors as well as, the size of the initial data. Here we consider a general Cauchy problem and present unified results, which apply to many special cases.
Keywords : Hyperbolic, Global Existence, Blow up, Large Gradient, Quasilinear.

1 Introduction

In this work we are concerned with the Cauchy problem for a one-dimensional first order quasilinear strictly hyperbolic system of the form

$$\begin{cases} u_t(x,t) = a(u(x,t), v(x,t))v_x(x,t) + f\ (u(x,t), v(x,t)) \\ v_t(x,t) = b(u(x,t), v(x,t))u_x(x,t) + g\ (u(x,t), v(x,t)), \end{cases} \tag{1.1}$$

where a subscript denotes a partial derivative with respect to the relevant variable, $x \in \mathbb{R}$, and $t > 0$.

Problems related to (1.1) have been discussed by many authors and results concerning global existence and formation of singularities in classical solutions have been established. For instance Lax [7] and MacCamy and Mizel [8] studied the system, for a depending on v only , $b \equiv 1$, $f \equiv g \equiv 0$, and showed that the solutions blow up in a finite time, even if the initial data are smooth and small. In his work, Lax required that $a' > 0$, at least in a neighbourhood of zero; whereas MacCamy and Mizel allowed a' to change sign. For a and b depending on u only, $f(u, v) =$ - $\lambda(u)v$, and $g \equiv 0$, Messaoudi [10] studied the propagation of heat guided by second sound and showed that classical solutions of a system, which is a special case of (1.1), break down in finite time, if the initial data are chosen small enough in the L^∞ norm but with large gradients. A similar system, where λ is taken constant, has been also discussed by Slemrod [13], Kosinski [6], Nishida [12] and blow up, as