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PERIODIC FIRST ORDER DELAY EQUATIONS WITH STATE DEPENDENT IMPULSES

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Abstract. Conditions are obtained for a first order nonlinear differential equation with delay and state dependent impulses to admit a periodic solution of bounded variation. The results are applied to the logistic equation with periodic intrinsic growth rate and periodic impulsive culling.
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1 Introduction and main results

Evolution processes subject to impulses, such as encountered in control theory and population dynamics, often give rise to a delay equation of the form

$$x'(t) + g(t-\tau, x(t-\tau)) = f(t) + \sum_{j=1}^{\infty} a_j(x)\,\delta_{t_j(x)}(t). \tag{1}$$

Some recent results on the existence and stability of solutions of this equation can be found in [2]-[5], [7] and [11] for the case where the moments of impulse $t_j(x)$ are independent of the state x, while in [1] and [8] results are obtained for cases when the moments of impulse are state dependent. Processes which are subject to T-periodic influences for a given $T > 0$ have also been studied in the case of state independent $t_j(x)$. For example, the global behavior of a periodic logistic system with periodic impulsive perturbations is analyzed in [9], while in [10] equation (1) is studied in the context of periodic boundary conditions. In this paper we show that under certain conditions, there exists a generalized T-periodic solution of (1) which is of bounded variation on $[0, T]$. The logistic equation with periodic intrinsic growth rate and culling is