SEPARATRIX SPLITTING THROUGH HIGH-FREQUENCY NON-SMOOTH PERTURBATIONS

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Abstract. Separatrices in integrable dynamical systems, if perturbed with analytic high frequency perturbations, split apart such that the splitting distance is exponentially small in the frequency parameter \( \omega \). This article utilizes a recent straightforward connection between the Melnikov function (which gives a measure of such a splitting) and a Fourier transform, to quantify such splitting under less smooth perturbations. If the perturbation is only piecewise \( C^k \) spatially, the splitting distance goes as \( \omega^{-k-1} \) for large \( \omega \).

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1 Introduction

A paradigm for the generation of chaotic motion is when an integrable two-dimensional system is subjected to a time-periodic perturbation. If the original system contained a heteroclinic orbit (a connecting trajectory between two fixed points), the generic picture is that this separatrix splits apart after perturbation, creating a heteroclinic tangle and consequent chaotic motion within that phase space region. The width of this tangled region therefore provides a good measure of the amount of chaoticity in the perturbed system. Many articles have established the exponential smallness of this width with respect to the perturbing frequency \([12, 21, 17, 13, 9, 16, 20, 8, 15]\). These usually relate to specific versions of the forced pendulum equation \([9, 16, 20, 8]\), analogous Hamiltonian systems \([12, 21, 17, 13]\), or else to specific assumptions on spatial analyticity of the perturbation \([15]\).

There have been few studies which assess the manifold splitting behavior under less smooth perturbations. The exceptions are \([14]\) which deals with near identity mappings and flows generating such mappings, and \([26]\) which examines a class of standard-like mappings. In each case – though not necessarily expressed in these words in these articles – the splitting distance has a reciprocal power dependence on the frequency, with the power depending on the smoothness.

This article provides a quick and easy derivation of this result in the general setting of continuous two-dimensional dynamical systems. The precise