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SEPARATRIX SPLITTING THROUGH HIGH-FREQUENCY NON-SMOOTH PERTURBATIONS

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Abstract. Separatrices in integrable dynamical systems, if perturbed with analytic high frequency perturbations, split apart such that the splitting distance is exponentially small in the frequency parameter ω. This article utilizes a recent straightforward connection between the Melnikov function (which gives a measure of such a splitting) and a Fourier transform, to quantify such splitting under less smooth perturbations. If the perturbation is only piecewise C^k spatially, the splitting distance goes as ω^{-k-1} for large ω.
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1 Introduction

A paradigm for the generation of chaotic motion is when an integrable two-dimensional system is subjected to a time-periodic perturbation. If the original system contained a heteroclinic orbit (a connecting trajectory between two fixed points), the generic picture is that this separatrix splits apart after perturbation, creating a heteroclinic tangle and consequent chaotic motion within that phase space region. The width of this tangled region therefore provides a good measure of the amount of chaoticity in the perturbed system. Many articles have established the exponential smallness of this width with respect to the perturbing frequency [12, 21, 17, 13, 9, 16, 20, 8, 15]. These usually relate to specific versions of the forced pendulum equation [9, 16, 20, 8], analogous Hamiltonian systems [12, 21, 17, 13], or else to specific assumptions on spatial analyticity of the perturbation [15].

There have been few studies which assess the manifold splitting behavior under less smooth perturbations. The exceptions are [14] which deals with near identity mappings and flows generating such mappings, and [26] which examines a class of standard-like mappings. In each case – though not necessarily expressed in these words in these articles – the splitting distance has a reciprocal power dependence on the frequency, with the power depending on the smoothness.

This article provides a quick and easy derivation of this result in the *general* setting of continuous two-dimensional dynamical systems. The precise