Dynamics of Continuous, Discrete and Impulsive Systems
Series **A:** *Mathematical Analysis* **14 (2007)** 321-332

EXISTENCE AND UNIQUENESS FOR NONLINEAR THIRD-ORDER TWO-POINT BOUNDARY VALUE PROBLEMS

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Abstract. The upper and lower solutions method, Leray-Schauder degree theory and differential inequality technique are employed to establish existence and uniqueness results for a class of nonlinear third-order two-point boundary value problems with one-sided Nagumo condition.

Keywords. One-sided Nagumo condition; upper and lower solutions method; Leray-Schauder degree theory

AMS (MOS) subject classification: 34B15

1 Introduction

Recently, M.D.R. Grossinho[1] established existence and location results for the third-order separated boundary value problems

$$x''' = f(t, x, x', x''), a < t < b,$$

with the following types of boundary conditions

$$x(a) = A,\ x''(a) = B,\ x''(b) = C$$

or

$$x(a) = A, c_1x'(a) - c_2x''(a) = B, c_3x'(b) + c_4x''(b) = C,$$

with $c_1, c_2, c_3, c_4 \in \mathbb{R}^+$and $A, B, C \in \mathbb{R}$.

In this work, we extend the study to the more general case as follows:

$$x''' = f(t, x, x', x''),\ a < t < b, \tag{1}$$

$$x(a) = A, \tag{2}$$

$$g(x'(a)) - [x''(a)]^p = B, \tag{3}$$

$$h(x(b), x'(b)) + [x''(b)]^q = C, \tag{4}$$

where $A, B, C \in \mathbb{R}, f(t, x, y, z) : [a, b] \times \mathbb{R}^3 \to \mathbb{R}$ is continuous, $g(y) : \mathbb{R} \to \mathbb{R}$ is continuous, $h(x, y) : \mathbb{R}^2 \to \mathbb{R}$ is continuous and decreasing on x, p, q are odd numbers.