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ON THE DIFFERENCE EQUATION $x_{n+1} = \frac{\alpha+\beta x_{n-1}+\gamma x_{n-2}+f(x_{n-1},x_{n-2})}{x_n}$

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Abstract. In this note we investigate the behavior of solutions of the difference equation

$$x_{n+1} = \frac{\alpha + \beta x_{n-1} + \gamma x_{n-2} + f(x_{n-1}, x_{n-2})}{x_n}, \qquad n = 0, 1, \dots$$

where α, β, γ and initial conditions x_{-2}, x_{-1}, x_0 are positive real numbers and $f : (0,\infty)^2 \to (0,\infty)$ is a real continuous function.

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1 Introduction

In this note we investigate the behavior of solutions of the difference equation

$$x_{n+1} = \frac{\alpha + \beta x_{n-1} + \gamma x_{n-2} + f(x_{n-1}, x_{n-2})}{x_n}, \qquad n = 0, 1, \dots \tag{1}$$

where α, β, γ and initial conditions x_{-2}, x_{-1}, x_0 are positive real numbers and $f : (0,\infty)^2 \to (0,\infty)$ is a real continuous function which satisfies some additional conditions which we shall explain later. We were motivated by the following conjecture (Conjecture 11.4.13 in [4]).

Conjecture 1. *Show that every positive solution of the equation*

$$x_{n+1} = \frac{1 + x_{n-1} + x_{n-2}}{x_n}, \quad n = 0, 1, \dots$$

converges to a two period solution.

Recently there has been a great interest in studying the periodic nature of nonlinear difference equations. For some recent results concerning, among other problems, the periodic nature of scalar nonlinear difference equations see, for example, [1-8] and references therein.

Our aim is to investigate some properties of positive solutions of equation (1). Among other results, we show that Conjecture 1 is not true.