

## ON A NONLINEAR SIZE-STRUCTURED PHYTOPLANKTON-ZOOPLANKTON AGGREGATION MODEL

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**Abstract.** We develop a nonlinear size-structured phytoplankton-zooplankton aggregation model. We establish a comparison principle and construct monotone sequences to show the existence of a weak solution. We also prove that this solution is unique. As an example, we construct a pair of upper and lower solutions for a large class of initial data to which all the theory presented applies.

**Keywords.** Phytoplankton-Zooplankton, Aggregation, Size-structured model, Existence-uniqueness

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### 1 Introduction

In this paper, we consider the following initial boundary value problem that describes the dynamics of a nonlinear size-structured phytoplankton-zooplankton (prey-predator) system

$$\begin{aligned}
 u_t + (g_1(x, t)u)_x &= \frac{1}{2} \int_0^x \beta(x-y, y)u(x-y, t)u(y, t)dy \\
 &\quad - \int_0^\infty \beta(x, y)u(x, t)u(y, t)dy \\
 &\quad - m_1(x, t, \varphi^u, \varphi^z)u && (x, t) \in (0, \infty) \times (0, T), \\
 z_t + (g_2(x, t)z)_x &= -m_2(x, t, \varphi^u, \varphi^z)z && (x, t) \in (0, \infty) \times (0, T), \\
 g_1(0, t)u(0, t) &= \int_0^\infty \gamma_1(y, t, \varphi^u, \varphi^z)u(y, t)dy && t \in (0, T), \\
 g_2(0, t)z(0, t) &= \int_0^\infty \gamma_2(y, t, \varphi^u, \varphi^z)z(y, t)dy && t \in (0, T), \\
 u(x, 0) &= u_0(x) && x \in (0, \infty), \\
 z(x, 0) &= z_0(x) && x \in (0, \infty),
 \end{aligned} \tag{1.1}$$