Dynamics of Continuous, Discrete and Impulsive Systems
Series **A:** *Mathematical Analysis* **14 (2007)** 251-264

POSITIVE SOLUTIONS FOR A CLASS OF SEMILINEAR ELLIPTIC SYSTEMS ON UNBOUNDED DOMAINS

Kuan-Ju Chen

Department of Applied Science, Chinese Naval Academy,
P.O. BOX 90175 Zuoying, Taiwan

Abstract. In this paper, we study the existence of positive solutions for a class of semilinear elliptic systems on some classes of unbounded domains.
Keywords. semilinear elliptic systems, positive solutions.
AMS (MOS) subject classification: 35J50, 35J55.

1 Introduction

This paper is devoted to the study of existence of positive solutions for a class of semilinear elliptic systems of the form:

$$\text{(I)}\quad \begin{cases} -\Delta u + u = g(v),\ \ u > 0 \ \text{ in } \ \Omega, \\ \\ -\Delta v + v = f(u),\ \ v > 0 \ \text{ in } \ \Omega, \end{cases}$$

where $(u, v) \in H_0^1(\Omega) \times H_0^1(\Omega)$ and Ω is an unbounded domain in $\mathbb{R}^N$.

The basic assumptions on the functions f and g are

(H1) f, $g \in C(\mathbb{R},\mathbb{R})$, with $f(t) = g(t) = 0$ for $t \leq 0$, $f(t) > 0$ and $g(t) > 0$ for $t > 0$. Both $F(t) = \int_0^t f(s)ds$ and $G(t) = \int_0^t g(s)ds$ are increasing and strictly convex in t.

(H2) $\lim\limits_{t\to 0^+} \frac{f(t)}{t} = 0$ and $\lim\limits_{t\to 0^+} \frac{g(t)}{t} = 0$.

(H3) There exists a constant $d > 0$ such that $f(t) \leq d(1 + t^p)$ and $g(t) \leq d(1 + t^q)$ for all $t \in \mathbb{R}^+$, where $1 < p$, $q < \frac{N+2}{N-2}$ if $N > 2$ and $1 < p < \infty$ if $N = 1$, 2.

(H4) There exists constants α, β with $p+1 < \alpha \leq 2p$ and $q+1 < \beta \leq 2q$ such that $0 < \alpha F(t) \leq tf(t)$ and $0 < \beta G(t) \leq tg(t)$ for $t > 0$.

Figueiredo and Yang [12] had shown that the existence of a ground state solution for the problem

$$-\Delta u + u = v^q, \ \ -\Delta v + v = u^p \ \text{ in } \ \mathbb{R}^N, \ \text{ where } 1 < p, q < \frac{N+2}{N-2},$$

by using spectral family theory of non-compact operator to find a suitable linking structure for the associated functional. In this paper, we establish