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ON SOLUTIONS FOR DIFFERENTIAL EQUATIONS IN BANACH SPACES VIA A NEW MEASURE OF WEAK COMPACTNESS

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Abstract. We establish the existence of solutions for a differential equation in Banach spaces. Our analysis relies on two approaches, one using the notion of ϕ-space together with a fixed point theorem for weakly sequentially continuous maps which are ϕ-condensing and the other using the Banach contraction principle.
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1 Introduction

In this paper we shall consider the following integral problem

$$u(t) = u_0 + \int_0^t f(t,u) \text{ on } I, \quad u_0 \in E, \tag{1}$$

where E is a Banach space, $I = [0,1]$ and f is a 1-parameter family of maps of E into E, i.e, $f : I \times E \to E$. The integral in (1) is understood to be the Pettis integral. It is well-known [4] that some classes of solutions of (1) solve the Cauchy problem

$$u' = f(t,u) \text{ on } I, \quad u(0) = u_0. \tag{2}$$

For the case $\|f(t,x)\| \leq M$ in a cylinder $P \subseteq \mathbb{R} \times E$, for instance, the existence of weak solutions for the corresponding problem over a reflexive space, first proved by Szep [1], solves (2). In this setting, O'Regan [2] proved an existence result for (1) assuming the following conditions:

(H_1) For each $u : I \to E$, the map $f(\cdot, u) : I \to E$ is Pettis integrable,

(H_2) For each $t \in I$, the map $f(t, \cdot) : E \to E$ is weakly sequentially continuous,