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THE 'OUTPUT STABILIZATION' PROBLEM: A CONJECTURE AND COUNTEREXAMPLE

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Abstract. 'Output stabilization' here refers to feedback which drives the system *output* to 0, without concern for the behavior of the full state. Since everything of concern is automatically observable, it is reasonable to conjecture — subject, of course, to some controllability hypothesis — that this output stabilization should always be possible by some kind of feedback from the output, with no necessity for the usual sort of observability hypothesis. This is true for the finite-dimensional case, but we show, by example, that the conjecture need not hold in infinite-dimensional contexts.
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1 Introduction

In discussing stabilization or feedback stabilizability of a system, standard analyses impose hypotheses of observability and controllability, from which is deduced stabilizability: that the *state* $x(\cdot)$ of the system is to go to the stationary point 0 as $t \to \infty$. We will here heuristically adopt the idea that, *"What we don't know can't hurt us"* — i.e., any deviation of the state which 'matters' to us must necessarily show up by affecting something we observe so, conversely, any deviation with no observable effects can be neglected as irrelevant. From this viewpoint, the stabilizability of what we *do* observe should be a more appropriate concern than seeking stabilizability of invisible components of the full state so we have a notion of **output stabilization**, meaning that the observable output y should go to 0 without direct concern for the behavior of the full state x.

Throughout we consider the autonomous linear control system

$$\dot{x} = \mathbf{A}x + \mathbf{B}u \qquad y = \mathbf{C}x \tag{1}$$

with the functions x, u, y taking values in spaces $\mathcal{X}, \mathcal{U}, \mathcal{Y}$, respectively and with suitable operators $\mathbf{A}, \mathbf{B}, \mathbf{C}$. We then denote the solution of (1) with initial data $\xi = x(0)$ by $x = x(\cdot; \xi, u)$ and the corresponding output $y = \mathbf{C}x$ by $y(\cdot; \xi, u)$.

Quite generally, we call a control system (1) **output stabilizable** if there is some mechanism to determine the control u dynamically (causally based