Dynamics of Continuous, Discrete and Impulsive Systems
Series **A:** *Mathematical Analysis* **14 (2007)** 287-299

EXPONENTIAL STABILITY OF EQUILIBRIA OF THE CURVE SHORTENING FLOW WITH CONTACT ANGLE

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Abstract. It is shown that mirror symmetric steady states of the evolution of three plane interfaces which move under the area preserving curve shortening flow and which meet in one single junction point are exponentially stable with respect to sufficiently small $C^{2+\alpha}$-perturbations.

Keywords. curve shortening flow, contact angle, triple junction, stable equilibria, sectorial operator.
AMS (MOS) subject classification: 35G30, 35K55, 35R35.

1 Introduction and main result

In this paper we study moving boundary value problems which occur in certain diffusion models of ternary alloys. There are particular situations in which the diffusion equations for the different phases in the bulk may be replaced by geometric evolution equations for the separating interphases only. Characteristic features of these evolution equations are the conservation of volume and the reduction of the interfacial energy. Therefore it is quite natural to consider the gradient flow of the surface functional under a fixed volume constrain. It is well-known that this gradient flow in L_2 is given by the averaged mean curvature flow in the case of an ambient dimension greater than two and the area preserving curve shortening flow in the plane, respectively.

Many papers are devoted to the study of the volume preserving mean curvature flow for compact surfaces without any boundary condition, see [5, 7, 3], and the references given therein. In contrast, moving boundary value problems for this flow have not been often investigated to date. In this paper we are interested in three-phase problems with fixed and moving boundary points. Due to the high complexity of the general situation we shall restrict our research to a mirror-symmetric configuration in the plane with a fixed contact angle at one end of the interfaces. The other ends are assumed to meet at a triple junction, satisfying there an angle condition and the so-called Young's law, cf. [14].