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PERIODIC SOLUTIONS OF NONLINEAR INTEGRODIFFERENTIAL EQUATIONS

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Abstract. This work provides sufficient conditions under which solutions of a class of nonlinear integrodifferential equations are a *priori* bounded. The topological transversality theorem is then applied to establish the existence of periodic solutions.
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1 Introduction

We are interested in the existence of periodic solutions of a nonlinear integrodifferential equation of the form

$$x'(t) = A(t)x(t) + \int_0^t f(t,s,x(s))ds + p(t) \tag{1.1}$$

where $t \in [0,\infty)$ denotes time, $'$represents derivative with respect to t, A is an $n \times n$ continuous T-periodic matrix function, $x(t) \in \mathbb{R}^n$. The input term $p : (0,\infty) \to \mathbb{R}^n$ is a continuous $T-$ periodic function.

The problem under investigation is the existence of T-periodic solutions of (1.1). This work is motivated by Burton *et al.* [3, 4] where they consider

$$f(t,s,x(s)) = C(t-s)x(s)$$

in the first paper and

$$f(t,s,x(s)) = k(t,s)g(x(s))$$

in the second paper. In both papers, the authors assume the existence of an *a priori* bound on possible solutions of a one parameter family of equations (the *a priori* bound being independent of the parameter) and then prove their result using the topological transversality theorem of Granas [7], where the details of this theorem can be found. The authors provide several examples where a variant of Liapunov's direct method is employed to obtain *a priori*