

## NETWORK OF NEURONS WITH DELAYED FEEDBACK: PERIODICAL SWITCHING OF EXCITATION AND INHIBITION

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**Abstract.** Considered is a class of periodic delay differential equations,

$$\dot{x}(t) = -a(t)x(t) + b(t)f(g(x_t)) + I(t),$$

which describes the dynamics of an isolated neuron in a periodic environment and also appears in several other applications. Applying the continuation theorem and the technique of Lyapunov functional, we establish some easily verifiable sufficient conditions on the existence and globally exponential stability of a periodic solution. These results greatly improve existing ones.

**Keywords.** Continuation theorem, delay differential equation, globally exponential stability, neurons, periodic solution.

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### 1 Introduction

In the last decades, neural networks have been intensively studied because of the successful hardware implementation and their various applications such as classification, associative memories, parallel computation, optimization, signal processing. These applications rely crucially on the analysis of the dynamical behavior of neural networks. To a large extent, the existing literature on theoretical studies of neural networks is predominantly concerned with autonomous systems containing temporally uniform network parameters and external input stimuli. Literature dealing with time-varying stimuli or network parameters appears to be scarce; such studies are however important to understand the dynamical characteristics of neuron behavior in time-varying environments.

Recently, Gopalsamy and Sariyasa [6, 7] considered a single neuron with delayed feedback such that the internal decay rate, the synaptic weight and stimulus are all periodically varying in time. The idealized model takes the form

$$\dot{x}(t) = -a(t)x(t) + b(t) \tanh[g(x_t)] + f(t), \quad (1)$$

where  $a, b, f \in C(\mathbb{R}, \mathbb{R})$  are periodic functions with a common period  $\omega (> 0)$ , and  $g(x_t)$  has one of the following forms: